

# Families among high-inclination asteroids

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## Abstract

We present a new classification of families identified among the population of high-inclination asteroids. We computed synthetic proper elements for a sample of 18,560 numbered and multi-opposition objects having sine of proper inclination greater than 0.295. We considered three zones at different heliocentric distances (inner, intermediate and outer region) and used the standard approach based on the Hierarchical Clustering Method (HCM) to identify families in each zone. In doing so, we used slightly different approach with respect to previously published methodologies, to achieve a more reliable and robust classification. We also used available SDSS color data to improve membership and identify likely family interlopers. We found a total of 38 *families*, as well as a significant number of *clumps* and *clusters* deserving further investigation.

*Key words:* Asteroids; Collisional physics; Asteroid dynamics.

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## 1 Introduction

As first realized by Hirayama (1918), some concentrations of asteroids are apparent if we look at their distribution in the space of orbital elements. These groups, known as the asteroid families, are believed to have originated from catastrophic disruptions of single parent bodies as a consequence of energetic asteroid collisions. These events are thought to have produced ejections of fragments into nearby heliocentric orbits, with relative velocities much lower than the parent body's orbital speed. Asteroid families were extensively investigated in the last decades, because these are unique *natural laboratories* to study the outcomes of high-energy collisions

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(Zappalà et al., 2002; Michel et al., 2003; Durda et al., 2007). Also, the number of currently identified families is an important constraint to model the collisional history of the asteroid main belt (Bottke et al., 2005).

Asteroid families are usually identified in the space of proper elements: proper semi-major axis ( $a_p$ ), proper eccentricity ( $e_p$ ), and proper inclination ( $I_p$ ). Proper orbital elements, being quasi-integrals of motion and thus nearly constant over time, are suited to be used to identify groupings that are stable and are not affected by transient oscillations of the osculating orbital elements.

To date, several tens of families have been discovered across the asteroid main belt (e.g. Zappalà et al., 1995; Bendjoya and Zappalà, 2002; Nesvorný et al., 2005). Most of these families are located at proper inclinations lower than about  $17^\circ$  ( $\sin(I_p) \leq 0.3$ ). The situation is more difficult at higher inclinations. The number of existing asteroids tends to decrease for increasing orbital inclination. As a consequence, asteroid surveys are usually centered around the ecliptic, and this also tends to introduce an observational bias against the discovery of high-inclination objects. Until recently, the number of known high-inclination asteroids was relatively small. Moreover, the number of high-inclination asteroids for which proper elements had been computed was even smaller. This was due to the fact that analytical proper elements (Milani and Knežević, 1990, 1994), which are computed for both numbered and multi-opposition asteroids, are not accurate enough for highly inclined orbits. In the past, computations of proper elements by means of the methods specially developed to handle highly inclined and/or eccentric orbits were carried out by some authors for a limited number of asteroids (Lemaitre and Morbidelli, 1994). More recently, the computation of the so-called synthetic proper elements (Knežević and Milani, 2000, 2003), has made it possible to compute with a good accuracy proper elements for high-inclination and high-eccentricity orbits as well. A large data set of asteroid proper elements is essential for the identification of asteroid families. Previous searches for families among high-inclination asteroids were seriously limited by the paucity of discovered asteroids and available proper elements. This problem affected also the only systematic search published in the recent years, namely that performed by Gil-Hutton (2006).

Recently, the number of known high-inclination asteroids has increased significantly. As of June 2010, when we commenced the present analysis, the database of synthetic proper elements, maintained at *AstDys* web page<sup>1</sup>, included 10,265 objects with  $\sin(I_p)$  greater than 0.295. In this database, however, synthetic proper elements were available only for numbered asteroids. In order to increase the available sample, following the approach described in Knežević and Milani (2003), we have computed synthetic proper elements for an additional sample of 8295 multi-opposition objects. In this way, for the purposes of our analysis, we have used a data-base of synthetic proper elements including 18,560 objects.<sup>2</sup> The distributions of these asteroids, in

<sup>1</sup> <http://hamilton.dm.unipi.it/astdys/index.php?pc=5>

<sup>2</sup> The accuracy of this proper element data set is similar to that of a recently analyzed sample of Hungaria asteroids (Milani et al., 2010). A slightly lower accuracy overall in proper eccentricity is likely due to the fact that our sample includes a

the  $(a_p, e_p)$  and  $(a_p, \sin(I_p))$  planes, are shown in Fig. 1. As a comparison, in his

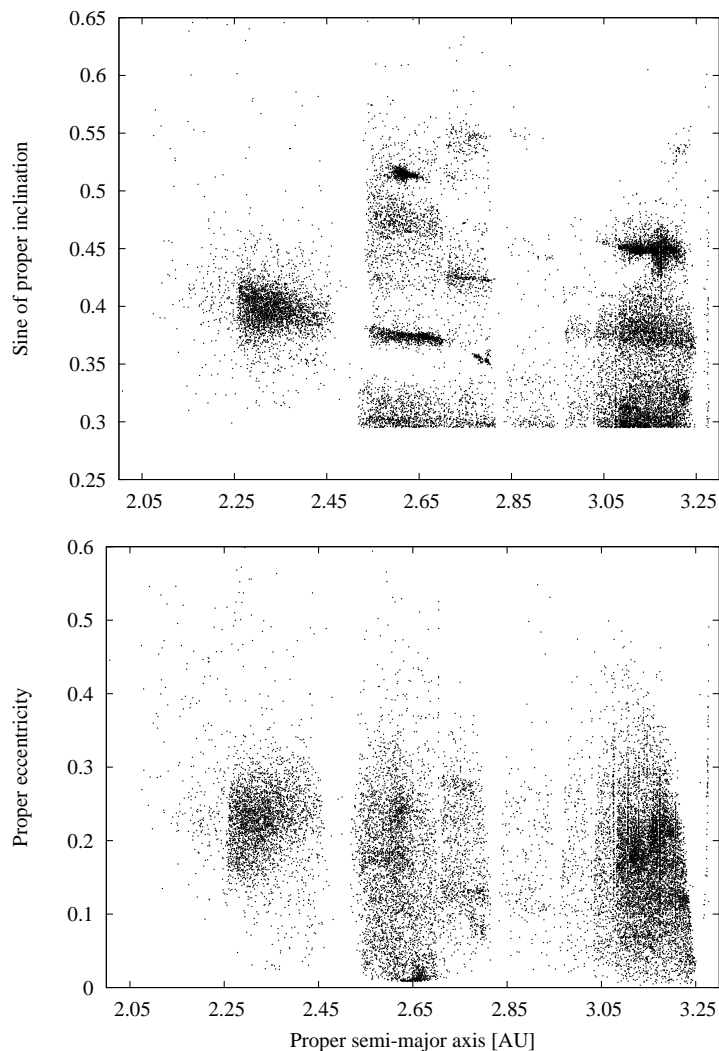


Fig. 1. The distribution of 18,560 highly inclined asteroids considered in the present analysis in the  $(a_p, \sin(I_p))$  (top) and  $(a_p, e_p)$  (bottom) planes.

search for high-inclination families Gil-Hutton (2006) used a sample about 5 times smaller (3697 asteroids).<sup>3</sup> Similar studies were also performed for the inner and intermediate regions of the asteroid belt by Carruba (2009, 2010), using only 1736 objects in the inner region, and 4452 objects in the intermediate zone. Our sample is about 2 times larger in the inner zone, and more than 20% larger in the intermediate zone (3553 and 5439 asteroids respectively). No search for families in the outer belt was performed so far in the investigations by Carruba.

larger number of highly eccentric orbits. The overall quality of the proper elements at our disposal is in any case fully appropriate for the purposes of our analysis.

<sup>3</sup> Instead of the limit of  $\sin(I_p)$  greater than 0.3, used by Gil-Hutton (2006), we chose to work with objects having  $\sin(I_p)$  greater than 0.295. This was done in order to identify possible traces of the classical main belt families among the highly inclined asteroids. In our sample there are 17,564 asteroids with  $\sin(I_p)$  greater than 0.3.

The purpose of this paper is to present a systematic search for families among high-inclination asteroids. The paper is organized as follows. In Section 2 the methods and techniques used to identify statistically significant groups of asteroids in the space of proper elements are described. The results of this analysis are presented in Section 3. These include a list of different kinds of groupings that we found in the inner, intermediate and outer part of the asteroid belt. In Section 4 we improve our analysis by taking into account the information about the object colors as obtained by the Sloan Digital Sky Survey (SDSS). Finally, in Section 5 we outline our main conclusions.

## 2 The Family Identification Method

In our analysis we have in general followed the approach described in the papers by Zappalà et al. (1990, 1994, 1995). Here we briefly summarize the main principles of this approach, we describe the main steps of its practical implementation, and explain a few modifications that we have introduced, mainly to improve the reliability of family membership.

We identified asteroid families in our proper element data-base by using the so-called Hierarchical Clustering Method (HCM) based upon the nearest-neighbor concept (see e.g. Zappalà et al., 1990). In general terms, the HCM approach is based on a few simple ideas. First, a metric is defined, to compute mutual distances between the objects in the space of proper elements. In particular, we have adopted the same metric that was used in previous papers. Therefore, the distance  $d$  between two objects is computed according to the relation:

$$d = na_p \sqrt{\frac{5}{4} \left( \frac{\delta a_p}{a_p} \right)^2 + 2(\delta e_p)^2 + 2(\delta \sin(I_p))^2} \quad (1)$$

where  $na_p$  is the heliocentric velocity of an asteroid on a circular orbit having the semi-major axis  $a_p$ .  $\delta a_p = a_{p1} - a_{p2}$ ,  $\delta e_p = e_{p1} - e_{p2}$ , and  $\delta \sin(I_p) = \sin(I_{p1}) - \sin(I_{p2})$ . The indexes (1) and (2) denote the two bodies under consideration. Note also that  $a_p$  in the above formula corresponds to the average of  $a_{p1}$  and  $a_{p2}$ .

With this choice,  $d$  has the dimension of a velocity, and is usually expressed in m/s. Once the mutual distances are computed, it is possible to identify the existence of groupings formed by objects that, at a given level of distance  $d$ , have distances from their closest neighbor smaller than  $d$ . The so-called *stalactite diagrams*, first used by Zappalà et al. (1990), are an effective way to display the groupings found at different distance levels, and to show how the membership of each group varies as a function of the distance limit. Examples will be given in the next section. Using this representation, the groupings of objects present in a given sample are graphically displayed as a system of *stalactites*, the most compact groupings being represented as the deepest stalactite branches.

The basic problem with the HCM approach is to define criteria for groupings that cannot be due to pure chance. The simple idea is that asteroid families produced by collisional processes should show up as deep and thick stalactite branches which cannot be produced by other mechanisms. To put this in more quantitative terms, in the classical papers adopting the HCM approach, a critical value of distance  $d_c$  was found, for which it could be reasonably concluded that groupings giving rise at deeper stalactite branches, or stalactites reaching the same distance level, but with unlikely high numbers of members, could not be due to chance, and necessarily have a physical origin.

In its practical implementation, the HCM approach includes therefore two basic parameters which have to be defined. One is the cut-off distance  $d_c$ . The second parameter is the minimum number of objects,  $N_{crit}$ , that is requested to characterize a statistically significant group (with respect to our selection criteria) at  $d_c$ .

As for  $d_c$ , in the previous papers this parameter was derived by creating artificial populations ("Quasi Random Populations") of synthetic objects, equal in number to the real population present in a given region of the proper element space, and built in such a way as to mimic independently the large-scale distributions of  $a_p$ ,  $e_p$ , and  $\sin(I_p)$  of the real population. The distance levels of the deepest stalactite branches formed by a minimum number of  $N_{crit}$  synthetic objects were recorded. By repeating this operation several times, an average value of minimum distance at which random grouping could still occur was derived, together with its uncertainty. This distance level was called Quasi Random Level (QRL), and was assumed to correspond to  $d_c$  for a given region of the proper element space. QRL built in such a way takes into account non-homogeneity of the distribution of the real objects in the given region. Thus, we can derive an evaluation of the distance level in that region for which we cannot expect that denser clusters of objects could exist purely due to chance. This takes into account the features of the non-homogeneous proper element distribution of the real objects in the given region. This approach is fairly powerful, but it is not totally exempt from problems. For instance, the obtained QRL is an average value of distance for a given region, and it "smears" out the small-scale, local properties of the distribution of objects. The main undesired consequence of this concerns the resulting definition of family memberships. The QRL concept is very satisfactory for the identification of families in a given region of the proper elements space, but it turns out to be a little too rigid as far as family membership is concerned, because the membership of a family tends to be more influenced by the local density of objects. For this reason, as we will explain below, in this paper we have modified the criteria adopted in previous application of the HCM for defining family memberships.

As for the adopted value of  $N_{crit}$ , it was chosen to be 5 at the epoch of the analysis performed by Zappalà et al. (1990). In subsequent analyzes considering increasingly bigger data-sets of asteroid proper elements (Zappalà et al., 1994, 1995), the adopted values of  $N_{crit}$  were scaled as the square root of the ratio between the numbers of objects in the newer and in the older sample. In particular, the values of  $N_{crit}$  used by Zappalà et al. (1995) were (10, 9, 8) for the inner, intermediate and outer zone respectively (for a definition of these zones, see later).

In the above-mentioned papers, it was clearly stated that the adopted values of  $d_c$  and  $N_{crit}$  had to be considered not as "solutions" of the problem, but as tentative guesses to be interpreted in a statistical sense. In particular, to adopt an identical  $d_c$  for all the families present in a given region of the proper element space, could lead to overestimate the membership of some families, and to underestimate the membership of some others.

In the present paper, we followed generally the same procedure, but we have introduced certain changes to take into account (1) some specificities of the high-inclination asteroid population; (2) some intrinsic limits of the statistical approach described above, mainly for what concerns the definition of the membership of identified families; (3) the fact that we know *a priori* that any asteroid sample cannot be complete beyond some value of magnitude, and family classifications tend to evolve as larger data sets of asteroid proper elements, corresponding to increasing number of discovered objects, become progressively available. In particular:

- In deriving the QRL, we did not remove *a priori* the members of big families possibly present in each zone, as was done by Zappalà et al. (1995). These authors did so to prevent the possibility that the presence of very populous families might affect the generation of the Quasi Random synthetic populations (by "saturating" some bins of the proper elements distributions). The reason why we did not implement this procedure in the present analysis is that the high-inclination population, mainly in the inner region, is notably non uniform, and the minimum distance level achieved by any population of fully-random synthetic objects would be in any case unreasonably high, leading to possible removal of very large fractions of the real objects present in some zone.
- We conservatively adopted as QRL not the average deepest level achieved by the stalactites of ten quasi-random populations generated in each zone, but the *deepest* level found in the ten cases.
- The membership of each family was derived not by looking simply at the objects present at QRL, but by making a more accurate analysis of each single case, as explained below.
- We paid attention to the possible presence of small, but very compact groupings which might be the cores of families consisting of small asteroids, most of which have not yet been discovered.

As a first step, we divided the main belt into three zones. They correspond to the inner, middle and outer region of the belt. The semi-major axis borders between adjacent zones are identical to those adopted by Zappalà et al. (1995), and correspond to the 3/1 and the 5/2 mean motion resonances (MMRs) with Jupiter. The  $a_p$ ,  $e_p$  and  $\sin(I_p)$  boundaries of the three regions are given in Table 1.

Next, for each zone, we defined a corresponding value of  $N_{crit}$ . In doing so, we tried to be as consistent as possible with the values previously used by Zappalà et al. (1995), taking into account the differences in both the numbers of objects present

in our regions, as well as the volumes of the regions themselves. In practical terms, however, we verified that the results of the family search are not very sensitive to the choice of  $N_{crit}$ . The adopted values are listed in Table 1.

Table 1

The properties of the three zones of asteroid belt considered in this work.

Parameter	Inner zone	Interm. zone	Outer zone
$a_{p_{min}}$ [AU]	2.065	2.501	2.825
$a_{p_{max}}$ [AU]	2.501	2.825	3.278
$e_{p_{min}}$	0.1	0.0	0.0
$e_{p_{max}}$	0.35	0.35	0.4
$\sin(I_{p_{min}})$	0.35	0.3	0.3
$\sin(I_{p_{max}})$	0.45	0.6	0.55
$N_{tot}$	3553	5439	9568
$N_{bins}$	8,3,3	8,3,3	6,3,3
$N_{crit}$	12	10	14
$QRL$ [m/s]	130	120	90

Having now at disposal a value of  $N_{crit}$  for each region, we could derive the corresponding QRL by applying the “classical” quasi-random population procedure described above. The resulting QRL values, together with the sets of discrete bins in the three proper elements adopted in each region, are also listed in Table 1.

Once the critical distance level is determined, we have all we need to perform in each zone our family identification task. We introduced thus some definitions, and we call *families* the groups whose stalactites reach at least  $QRL - 10$  m/s, with a number of members larger or equal to  $N_{crit}$ , or reach only QRL, but having at that level a number of members equal to at least  $N_{crit} + 2\sqrt{N_{crit}}$ . As an additional requirement, we impose that a family must in any case be found at QRL, and must be separated from all other groups existing at that distance level.

We call then *clumps* the groups which marginally fail the above criteria for family classification. These are groups whose stalactite reaches QRL, with a number of members larger than  $N_{crit}$  but smaller than  $N_{crit} + 2\sqrt{N_{crit}}$ , or groups which reach  $QRL - 10$  m/s, with  $N_{crit}$  members, but merge with some other group at QRL. We stress that what we call clumps are *not* fully flagged families. Many of them are produced by families which split just below the QRL. In principle, these groups may be interesting, since it is not clear *a priori* whether they may simply represent the outcome of the “erosion” of families or they may correspond to some physical process, including secondary fragmentation.

The above definition of families, which is strictly related to the concepts of QRL and  $N_{crit}$ , is valid in statistical terms only. In particular, there is a risk that we might miss some groupings which fail to satisfy the above criteria, but may well have a physical origin. As an example, very compact, but small groups, possibly including many objects too small to have been discovered so far, could produce in stalactite diagrams very deep stalactite branches, but too thin to satisfy our family definition requirements. Another case is that of groupings originating within nominal

families as the outcomes of some possible second-generation disruption event, but being, again, too small to produce stalactite branches satisfying the above family classification criteria.

The term *cluster* has been used in the past to describe small and very compact asteroid groupings which are clearly distinct from the background (see e.g. Farinella et al., 1992; Cellino et al., 1993; Zappalà et al., 1994; Nesvorný et al., 2002a). Accordingly, we define here as *clusters* the groupings whose corresponding stalactites may well be noticeably compact and deep to suggest a possible physical origin, though not formally satisfying our above definition of families or clumps. We prefer here to be flexible and we do not introduce a more rigid definition of what we call clusters. In the next Section, we list for each zone the number of clusters which we identified from a subjective analysis of our HCM results. Of course, some arbitrariness can be present, but we think that in many cases the clusters that we found interesting indeed deserve further investigation.

We note that the family classification criteria described above are the result of several experiments performed using slightly different options. For example, we investigated how different requested separation between families would influence the results. We found that the final classification is not very sensitive to small changes in our requirements. In particular, such changes have very modest consequences on the resulting list of families in the intermediate and outer zones. Most families in these zones tend to be quite robust, and are identified by adopting a large variety of possible criteria. The situation is somewhat different in the inner zone, where the list of families may change significantly by using different classification criteria. This is due to the specific situation in this region, which is dominated by one large group.

As discussed above, the concept of QRL is quite useful to identify significant groupings. On the other hand, it has also some drawbacks as far as the determination of family membership is concerned. Different families may well be characterized by differences in the original events that produced them, may have different ages, may have experienced therefore different evolutions since their birth, and may be immersed in different environments in the space of proper elements. As a consequence, in this paper we have adopted a case-by-case approach, proposed by Nesvorný et al. (2005), to better define the most likely membership of each identified family.

As an example of this approach, we present here our analysis for the case of the grouping around the asteroid (116763) 2004EW<sub>7</sub> located in the intermediate region. The procedure described below is fully representative of what we did in general, with only a few exceptions. In Fig. 2 we show the resulting number of members as a function of the adopted distance cut-off level  $d_c$ . After an initial growth at small distance levels, it is apparent the presence of a *plateau*, an interval of distance at which the family membership remains nearly constant, before further growth due to incorporation of some neighboring group, and a final merging with the background. The plateau extends from 70 to 150 m/s. In such cases, we choose as the most appropriate distance level for defining membership the one corresponding to the center of the plateau. When two or more plateaus are present (e.g. as in the case of the



Tina family) usually we adopt the value corresponding to the center of the deepest plateau (the one occurring at smaller distance). As for the definition of plateau center, whenever the plateau consists of an even number  $n$  of points (we record membership at discrete steps of 10 m/s in distance), we conservatively adopted the  $n/2$ th smallest distance level in the plateau to define family membership. Whenever the number  $n$  is odd, we simply choose the central distance value in the plateau.

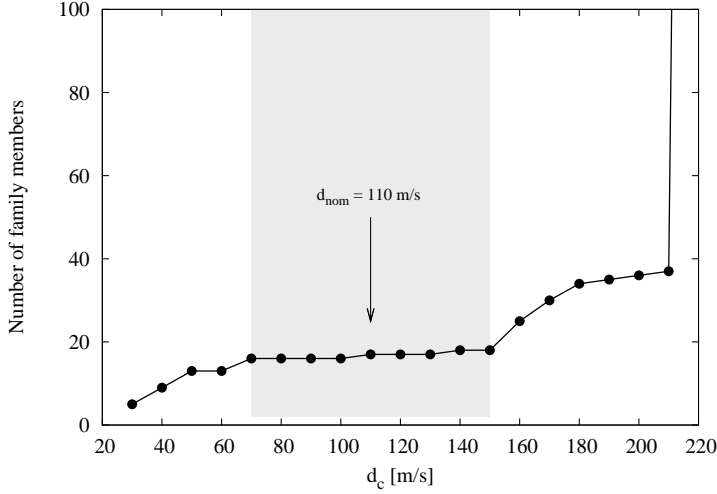


Fig. 2. The number of asteroids belonging to the family around the asteroid (116763) 2004EW<sub>7</sub>, as a function of the distance cut-off level  $d_c$ . It can be seen that below 70 m/s the core of the family is seen to grow. At 70 m/s the growth nearly stops and the number of members remains almost constant until 150 m/s. At 160 m/s some limited growth occurs again, until 210 m/s when family starts to merge with the background population in the surrounding region.

In a few cases of very complex families (e.g. Euphrosyne) the above procedure cannot be applied. In these cases we analyzed the structure of the family, and decided subjectively our preferred distance level for family membership.

The same method was also used to determine the membership of clumps. In the case of the clusters, we did not devise any special criterion to define membership, but we simply looked subjectively at their stalactite structure, taking profit of the fact that these groups are very compact over large intervals of distance level.

Before presenting the results of our application of the HCM, let us note that, as we will show in Section 4, we have complemented our proper element analysis by taking advantage also of some additional input, namely the evidence coming from available SDSS color data for the objects of our sample. This allowed us to improve our interpretation of proper element data, opened the possibility to check the consistency of HCM-based family classification in terms of likely mineralogical composition, and allowed us to identify in some cases possible random interlopers.

### 3 Summary of HCM results

In this section we present and discuss the results of our HCM-based analysis. We split our discussion into three separate sub-sections, devoted to groupings identified in the inner, intermediate and outer zone, respectively. The positions of identified asteroid families in the  $(a_p, e_p)$  and  $(a_p, \sin(I_p))$  planes are shown in Figs. 3 and 4.

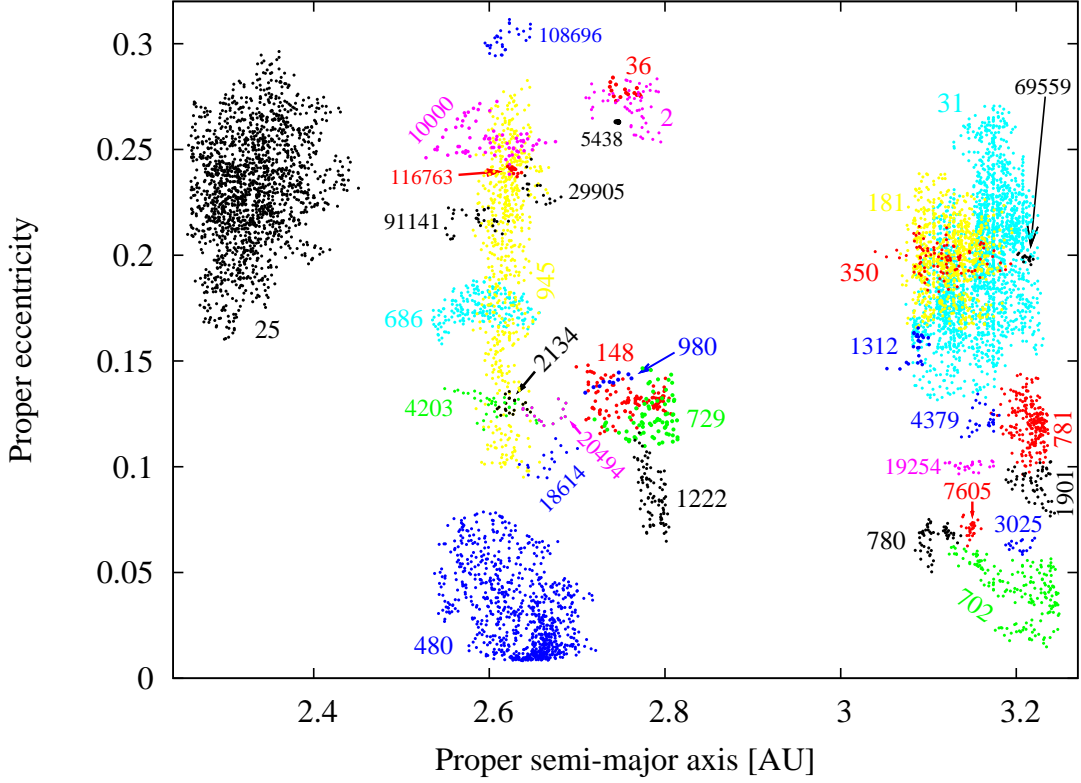


Fig. 3. Locations of the asteroid families, identified in this work, in the  $(a_p, e_p)$  plane. Different types and sizes of symbols are used to distinguish between the members of different families.

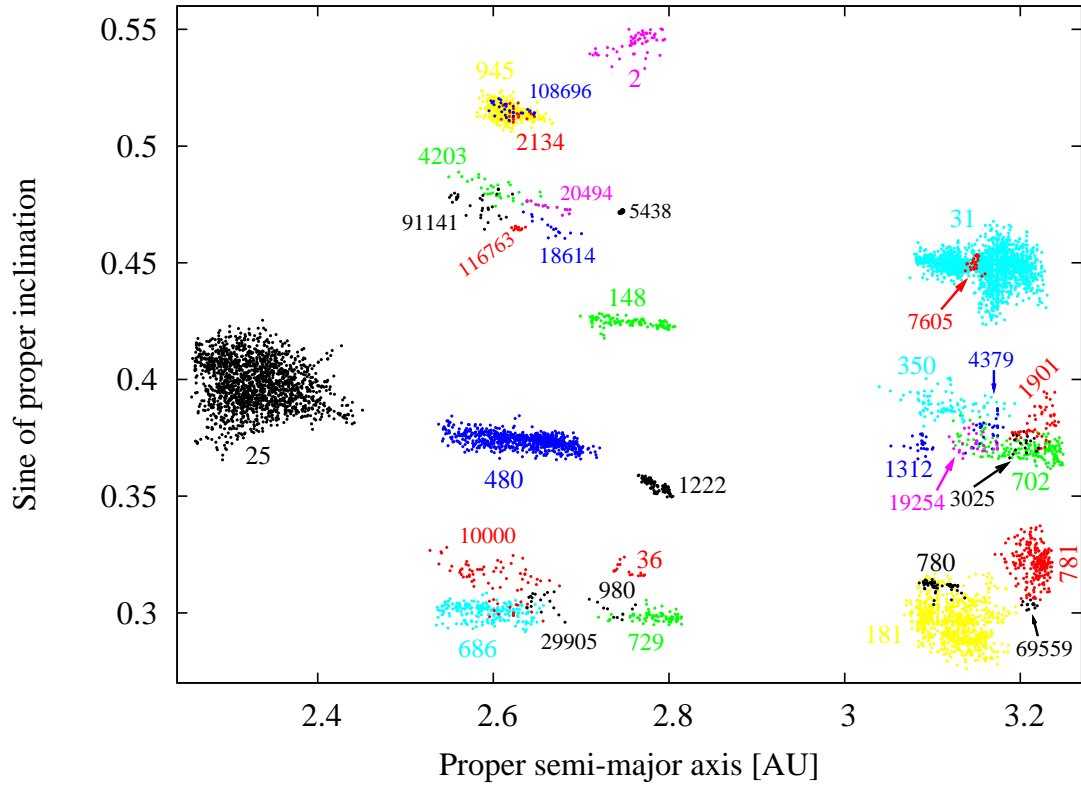


Fig. 4. The same as Figure 3, but in the  $(a_p, \sin(I_p))$  plane.

The lists of families, clumps and clusters are given in Tables 2,3 and 4, respectively.<sup>4</sup>

<sup>4</sup> The complete list of members of all identified families can be found at:  
<http://poincare.matf.bg.ac.rs/~bojan/asteroids/families/high-i-fam/list.html>

Table 2: List of the identified asteroid families. For each group, the columns give the lowest-numbered member; the smallest distance level  $d_{min}$  at which it includes  $N_{crit}$  objects; the adopted distance level  $d_{nom}$  for membership; the resulting total number  $N$  of members; the proper elements  $a_p$ ,  $e_p$  and  $\sin(I_p)$  of the lowest-numbered member.

Name	$d_{min}$	$d_{nom}$	$N$	$a_p$	$e_p$	$\sin(I_p)$
Inner zone						
(25) Phocaea	60	120	1694	2.400	0.228	0.397
(7784) 1994PL	110	120	19	2.268	0.197	0.418
Intermediate zone						
(2) Pallas	100	120	57	2.771	0.281	0.548
(36) Atalante	120	120	16	2.749	0.275	0.324
(148) Gallia	40	120	113	2.771	0.132	0.425
(480) Hansa	20	110	839	2.644	0.009	0.375
(686) Gersuind	60	100	207	2.589	0.173	0.302
(729) Watsonia	50	160	139	2.760	0.123	0.299
(945) Barcelona	40	130	600	2.637	0.251	0.512
(980) Anacostia	110	130	18	2.741	0.140	0.298
(1222) Tina	50	120	89	2.793	0.082	0.354
(2134) Dennispalm	90	110	19	2.638	0.130	0.512
(4203) Brucato	100	130	46	2.605	0.132	0.483
(10000) Myriostos	110	150	73	2.587	0.269	0.319
(18614) 1998DN <sub>2</sub>	120	120	16	2.644	0.100	0.470
(20494) 1999PM <sub>1</sub>	110	110	14	2.684	0.127	0.473
(29905) 1999HQ <sub>11</sub>	120	140	28	2.675	0.226	0.299
(89713) 2001YB <sub>113</sub>	110	110	11	2.578	0.092	0.369
(91141) 1998LF <sub>3</sub>	100	150	30	2.599	0.222	0.474
(108696) 2001OF <sub>13</sub>	90	130	36	2.647	0.309	0.514
(116763) 2004EW <sub>7</sub>	50	50	13	2.625	0.240	0.465
Outer zone						
(31) Euphrosyne	30	100	2063	3.155	0.208	0.447
(181) Eucharis	50	90	373	3.128	0.217	0.305
(350) Ornamenta	80	110	93	3.114	0.192	0.387
(702) Alauda	70	90	179	3.194	0.021	0.369
(780) Armenia	30	120	76	3.117	0.070	0.312
(781) Kartvelia	30	80	232	3.227	0.103	0.312
(1312) Vassar	90	100	24	3.094	0.161	0.370
(1444) Pannonia	80	100	18	3.158	0.140	0.323
(1901) Moravia	70	80	54	3.237	0.097	0.389
(3025) Higson	70	80	17	3.207	0.059	0.374
(4379) Snelling	80	90	29	3.168	0.120	0.370
(5931) Zhvanetskij	70	90	64	3.192	0.164	0.304
(7605) 1995SR <sub>1</sub>	60	120	30	3.151	0.071	0.453
(19254) 1994VD <sub>7</sub>	80	80	26	3.160	0.101	0.370
(52734) 1998HV <sub>32</sub>	80	80	16	3.101	0.140	0.451
(69559) 1997UG <sub>5</sub>	60	60	14	3.214	0.198	0.304

Table 3: The same as in Table 2, but for the identified clumps. The smallest distance level  $d_{min}$  corresponds to  $N_{crit}/2$

Name	$d_{min}$	$d_{nom}$	$N$	$a_p$	$e_p$	$\sin(I_p)$
Inner zone						
(2745) SanMartin	90	120	22	2.288	0.159	0.386
(26142) 1994 $PL_1$	110	120	13	2.264	0.176	0.385
(100681) 1997 $YD_1$	110	110	10	2.278	0.266	0.419
Intermediate zone						
(194) Prokne	110	120	18	2.617	0.196	0.296
(2382) Nonie	80	110	19	2.760	0.275	0.544
(4404) Enirac	50	110	52	2.644	0.113	0.512
(40134) 1998 $QO_{53}$	80	150	24	2.735	0.226	0.433
(59244) 1999 $CG_6$	110	120	11	2.634	0.165	0.471
(62074) 2000 $RL_{79}$	50	110	33	2.586	0.091	0.372
(81583) 2000 $HD_{46}$	50	100	44	2.616	0.162	0.512
(103219) 1999 $YX_3$	80	110	13	2.642	0.082	0.371
(114822) 2003 $ON_{15}$	70	110	24	2.740	0.139	0.424
(195207) 2002 $DN_2$	90	100	5	2.565	0.114	0.479
Outer zone						
(1101) Clematis	50	70	16	3.242	0.034	0.369
(1612) Hirose	60	70	20	3.102	0.115	0.307
(2793) Valdaj	70	80	45	3.164	0.076	0.378
(2967) Vladisvyat	60	80	74	3.210	0.116	0.296
(13935) 1989 $EE$	70	70	10	3.140	0.261	0.450
(14424) Laval	80	80	14	3.145	0.118	0.371
(15161) 2000 $FQ_{48}$	80	100	25	3.203	0.173	0.338
(16243) Rosenbauer	90	100	25	3.149	0.155	0.331
(22805) 1999 $RR_2$	60	70	17	3.147	0.171	0.304
(23886) 1998 $SV_{23}$	80	80	16	3.128	0.110	0.309
(25295) 1998 $WK_{17}$	80	90	19	3.174	0.105	0.383
(26324) 1998 $VG_{16}$	60	90	19	3.129	0.035	0.380
(28884) 2000 $KA_{54}$	90	90	18	3.093	0.038	0.373
(29596) 1998 $HO_{32}$	80	80	22	3.142	0.142	0.297
(34676) 2000 $YF_{126}$	60	70	15	3.211	0.157	0.297
(35664) 1998 $QC_{64}$	90	90	14	3.112	0.063	0.371
(38834) 2000 $SP_1$	80	80	16	3.125	0.199	0.386
(52661) 1998 $BT_8$	80	90	18	3.109	0.072	0.373
(55940) 1998 $GU_8$	80	80	27	3.170	0.157	0.435
(58892) 1998 $HP_{148}$	60	70	18	3.135	0.162	0.305
(71193) 1999 $XG_{231}$	80	90	18	3.091	0.163	0.308

Table 4: The same as in Table 2, but for proposed asteroid clusters. The smallest distance level  $d_{min}$  corresponds to  $N_{crit}/2$

Name	$d_{min}$	$d_{nom}$	$N$	$a_p$	$e_p$	$\sin(I_p)$
Inner zone						
(2860) Pasacentennium	100	120	9	2.332	0.161	0.392
(6246) Komurotoru	120	120	11	2.447	0.255	0.396
(31359) 1998 $UA_{28}$	80	100	11	2.272	0.200	0.403
(58419) 1996 $BD_4$	80	110	10	2.276	0.233	0.369
Intermediate zone						
(247) Eukrate	90	110	5	2.741	0.202	0.427
(5438) Lorre	10	50	8	2.747	0.263	0.472
(36240) 1999 $VN_{44}$	90	110	5	2.619	0.185	0.466
(44219) 1998 $QB_3$	60	110	7	2.724	0.121	0.510
(48606) 1995 $DH$	90	100	5	2.668	0.109	0.478
(76404) 2000 $FG_{13}$	70	80	6	2.623	0.191	0.299
(91136) 1998 $KK_6$	70	110	6	2.613	0.145	0.483
(103056) 1999 $XX_{134}$	90	100	9	2.623	0.281	0.512
(109195) 2001 $QE_{75}$	80	90	7	2.656	0.084	0.373
(208080) 1999 $VV_{180}$	70	90	6	2.608	0.119	0.513
Outer zone						
(24440) 2000 $FB_1$	40	50	16	3.167	0.171	0.437
(30575) 2001 $OM_{101}$	60	60	11	3.124	0.045	0.380
(59853) 1999 $RP_{82}$	50	100	14	3.044	0.099	0.322
(63530) 2001 $PG_{20}$	90	160	53	2.888	0.111	0.300

### 3.1 Inner zone

In the inner belt region (2.065 – 2.501 AU) we analyzed a sample of 3553 numbered and multi-opposition asteroids with  $\sin(I_p) \geq 0.295$ . These objects are separated from low-inclination main belt asteroids belt and from the neighboring Hungaria region by both mean-motion and secular resonances (Knežević and Milani, 2003; Carruba, 2009). An inner boundary in semi-major axis, often related to the 7/2 mean motion resonance with Jupiter, is located at about 2.25 AU. However, a non-negligible number of asteroids are still present beyond this limit. The outer boundary, located close to 2.5 AU, is set by the powerful 3/1 MMR with Jupiter. Moreover, the region is delimited by important secular resonances (SRs): the  $\nu_6 = g - g_6$  at low inclination, and the  $\nu_5 = g - g_5$  and  $\nu_{16} = s - s_6$  at high inclination<sup>5</sup> (Knežević et al.,

<sup>5</sup> The  $g$  and  $s$  are the average rates of the longitude of perihelion  $\varpi$  and of the longitude of node  $\Omega$ , respectively. The indexes 5 and 6 refer to planets Jupiter and Saturn, respectively.

1991; Michtchenko et al., 2010). Although deep close encounters with Mars are not possible due to the Kozai class protection mechanism (Milani et al., 1989), even shallow encounters may result in removal of asteroids for eccentricities higher than about 0.3. These are the most important dynamical mechanisms which separate the Phocaea group from the rest of the main belt. Therefore, these high-inclination asteroids seem to be located in a stability island. Since they are filling up a bounded stability region and are quite concentrated, this makes any search for collisional families rather difficult.

Using our procedures, we identified a couple of nominal families, and a small number of clumps. In addition, we found also several potentially interesting smaller groupings which are classified as clusters.

The region is dominated by one single, large group of asteroids, whose lowest-numbered object is (25) Phocaea. Since this group contains almost 50% of objects in the region, the identification of other significant groupings is quite complicated. This is very similar to the situation that is found in the Hungaria region (Warner et al., 2009; Milani et al., 2010).

In Fig. 5 the resulting stalactite diagram for our sample is shown. As can be seen, the dominant feature is the large group of (25) Phocaea. Many minor groups visible in the diagram are substructures of it, and if Phocaea is a real collisional family, at least some of its subgroups could represent the outcomes of second-generation collisions. If this is true, it is likely that Phocaea is an old family, as was recently suggested by Carruba (2009) who estimated it to be up to 2.2 Gyr old.

Apart from Phocaea, we identified only one other nominal family according to our selection criteria. This is a group whose lowest-numbered object is (7784) 1994PL. This family is identified here for the first time.

Two families proposed by Gil-Hutton (2006), namely Wood and Krylov are not confirmed. There is a grouping including asteroid (1660) Wood whose lowest-numbered object is (1192) Prisma, but this group did not pass our significance criteria. We did not find any significant grouping associated with asteroid (5247) Krylov. This asteroid is a member of the Phocaea family at a distance level of 120 m/s.

Moreover, a grouping including (2860) Pasacentennium, which was previously found by Gil-Hutton (2006), is fairly small, and we classify it now tentatively as a cluster, although its stalactite branch is not very deep. Similarly, a grouping around the asteroid (6246) Komurotoru classified as a clump by Carruba (2009), who performed this search in the space of proper frequencies (see Carruba and Michtchenko (2007) for details on this methodology), is included in our list of clusters. Carruba (2009) classified (26142) 1994PL<sub>1</sub> as a clump, and this is confirmed by our analysis.

None of the other groups proposed by Gil-Hutton (2006) and Carruba (2009) have passed our significance criteria. Among these groups there is also (19536) 1999JM<sub>4</sub>, classified as a family by Carruba (2009).

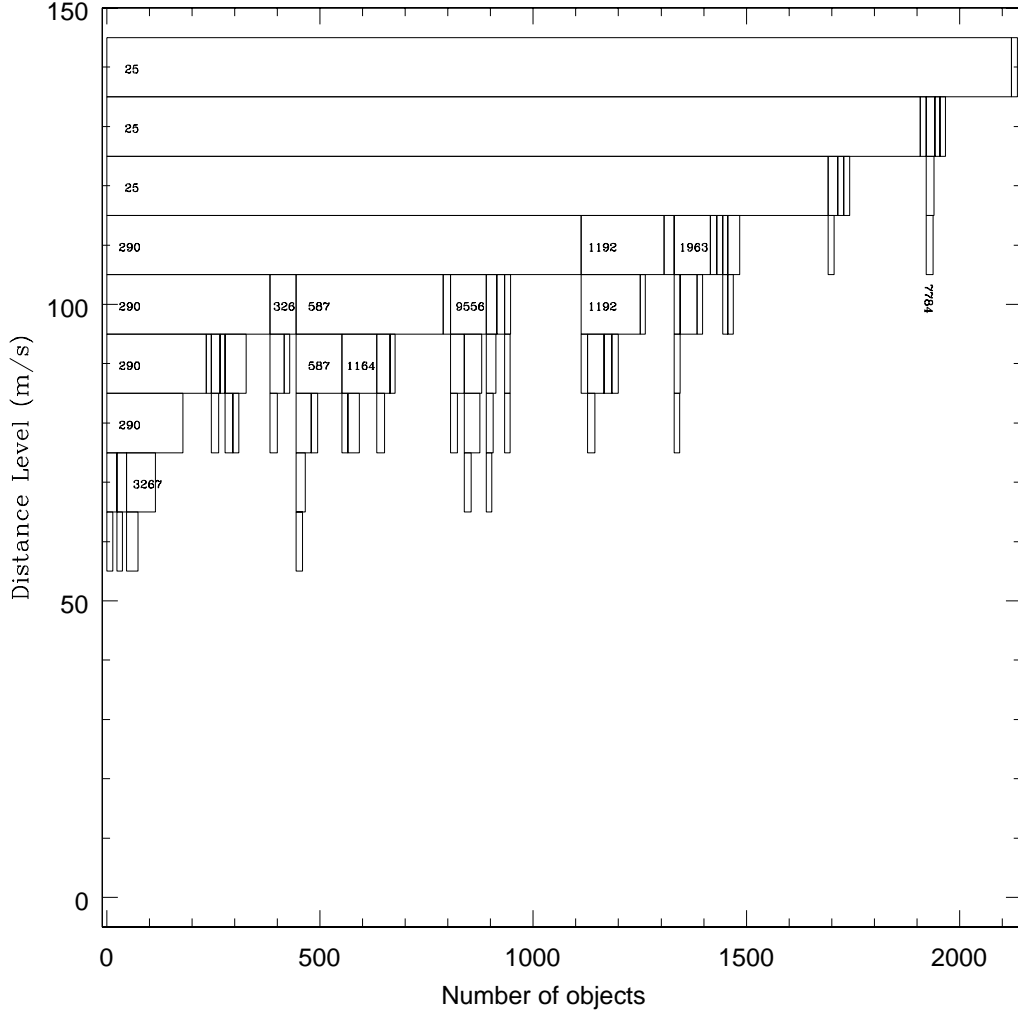


Fig. 5. The stalactite diagram for the inner zone ( $N_{crit}=12$ ,  $QRL = 130$  m/s). At each distance level, only groupings having at least  $N_{crit}$  members are plotted.

### 3.2 Intermediate zone

In our high-inclination sample, 5439 asteroids belong to the intermediate zone (2.501 – 2.825 AU). This region is characterized by a roughly uniform distribution of objects in the semi-major axis *versus* eccentricity plane, whereas concentrations and gaps are apparent in the semi-major axis *versus* inclination plane (see Fig. 1). From a dynamical point of view, this zone is characterized by a mixing of stable and chaotic regions. The eight stable islands are separated by three mean motion (Jupiter-asteroid) and three linear secular resonances (see Carruba, 2010).

The stalactite diagram for this zone is shown in Fig. 6. We found 19 asteroid families, 10 clumps and 10 clusters.



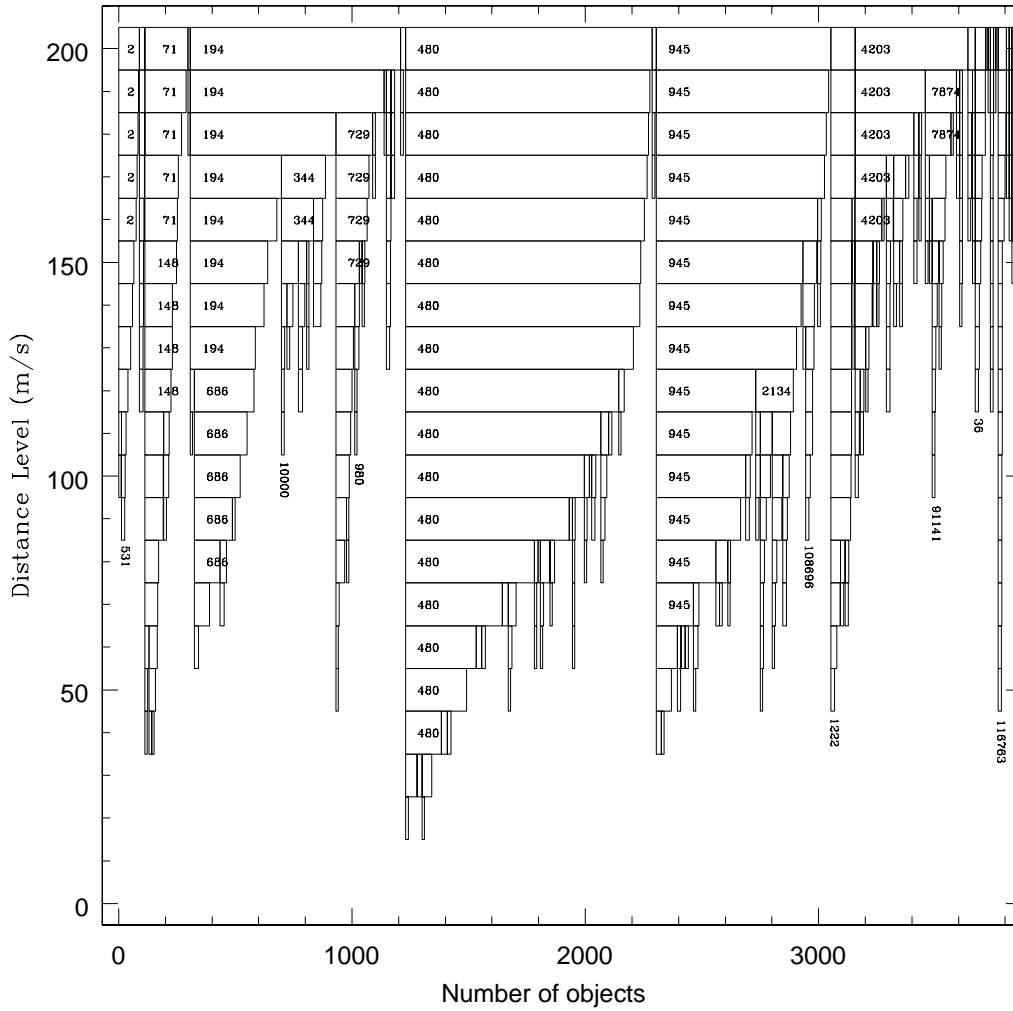


Fig. 6. The stalactite diagram for the intermediate zone ( $N_{crit}=10$ ,  $QRL = 120$  m/s). As in Fig. 5, at each distance level only groupings having at least  $N_{crit}$  members are shown.

As can be seen in Fig. 6, the situation in this region is very different from what is found in the inner region. Several important groupings are immediately recognizable. Two large families are prominent: these are the families of (480) Hansa and (945) Barcelona. They were already mentioned in the literature. The Hansa family was originally proposed by Hergenrother et al. (1996), while a Barcelona family was first identified by Foglia and Masi (2004). Both families were also confirmed later by other authors (e.g. Gil-Hutton, 2006; Carruba, 2010). In addition to Hansa and Barcelona, however, several sharp and deep stalactite branches can be seen in Fig. 6, corresponding to families whose collisional origin seems very likely.

The (686) Gersuind family, first identified by Gil-Hutton (2006) is confirmed, while another group originally classified as a clump by the same author, Gallia, is now a full-flagged family.

The families of (1222) Tina and (4203) Brucato, recently proposed by Carruba (2010), are confirmed as well.<sup>6</sup> It is interesting to note that Carruba (2010) identified Brucato family in the space of proper frequencies only, while in the space of proper elements he identified it as a clump. We also confirm the existence of a Watsonia family, mentioned by Cellino et al. (2002) on the basis of spectroscopic properties pointed out by Burbine et al. (1992) and Bus (1999). According to still unpublished observations (Cellino 2011, in preparation) (729) Watsonia belongs to a rare group of objects, called *Barbarians* after their prototype, the asteroid (234) Barbara, which exhibit unusual polarimetric properties (Cellino et al., 2006; Masiero and Cellino, 2009). Very interestingly, we found in this region another family, whose lowest-numbered member is (980) Anacostia, which is also a Barbarian (Gil-Hutton et al., 2008). The Watsonia and Anacostia families merge together well above the QRL.

In addition, we also found 10 new families, having as their lowest-numbered objects (36) Atalante, (2134) Dennispalm, (10000) Myriostos, (18614) 1998DN<sub>2</sub>, (20494) 1999PM<sub>1</sub>, (29905) 1999HQ<sub>11</sub>, (89713) 2001YB<sub>113</sub>, (91141) 1998LF<sub>3</sub>, (108696) 2001OF<sub>13</sub>, and (116763) 2004EW<sub>7</sub>.

The family proposed by many authors (see e.g. Williams, 1992; Lemaitre and Morbidelli, 1994; Gil-Hutton, 2006; Carruba, 2010) around the very large asteroid (2) Pallas passed our selection criteria as well. A group including Pallas is present at  $QRL - 10$  m/s, but it consists of 8 members, only. However two other groupings associated to asteroids (531) Zerlina and (1508) Kemi, which have 14 and 23 members at  $QRL - 10$  m/s respectively, merge with group around Pallas at QRL forming a group of 57 asteroids.

The Pallas family is certainly interesting in terms of possible composition, since (2) Pallas belongs to a fairly rare taxonomic class (*B*). According to Clark et al (2010) (2) Pallas is the largest object belonging to a small number of *B*-class asteroids which exhibit a blueish trend in the reflectance spectrum which extends also in the near-IR. No other asteroid which has been found so far to share this same behavior belongs to our family. However, we do know that several members of the family are classified as *B*-class, as pointed out by Clark et al (2010). In this respect, the member list that we find now is largely in agreement with that given by the above authors. Spectroscopic observations extending into the near-IR of members of the Pallas family will be very interesting to confirm a genetic relationship with (2) Pallas. Since (2) Pallas is one of the biggest asteroids (it is actually the biggest one, if (1) Ceres is considered to be a dwarf-planet) its family could well be another example of the outcome of an energetic cratering event, as in the well known case of Vesta. If this is true, it is likely that many members are quite small and faint (Pallas being a low-albedo object), and have not yet been discovered. Present and

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<sup>6</sup> In the cases of families like Tina, whose members interact with one or more secular resonances, the proper elements we used are not fully appropriate and identification results might be different if resonant proper elements (Lemaitre and Morbidelli, 1994; Carruba and Morbidelli, 2011) would be used. This, however, seems more important for family membership than for recognition of family.

future sky surveys will hopefully be able to confirm or reject this hypothesis.

In the intermediate zone we have also found numerous clusters. Two of these, namely (5438) Lorre and (44219) 1998 $QB_3$  are extremely compact. Both clusters are clearly distinct from any other grouping, and remain separated even at very large distance levels around 200 m/s. These facts suggest a real collisional origin for these clusters. Moreover, it is known that size and shape of asteroid families change over time, with respect to the original post-impact situations. Families slowly spread, and became more and more dispersed due to the chaotic diffusion and gravitational and non-gravitational perturbations (Bottke et al., 2001; Nesvorný et al., 2002b; Carruba et al., 2003; Dell’Oro et al., 2004). Being so compact, it is also likely that the two above-mentioned clusters should be quite young. We are currently carrying out a detailed study of these and other clusters, to be presented in a separate paper.

### 3.3 Outer zone

Our sample includes 9568 high-inclination asteroids in the outer zone (2.825 – 3.278 AU), but most objects are located at  $a_p \gtrsim 3.05$  AU. Similarly to the case of the intermediate zone, the distribution of objects in the outer region is roughly uniform in the semi-major axis *versus* eccentricity plane, whereas in the semi-major axis *versus* inclination plane most asteroids are concentrated in three different dominions. One of them is located close to  $\sin(I_p) = 0.3$ , another one is centered around  $\sin(I_p) = 0.38$ , while the third one is centered around  $\sin(I_p) = 0.45$  (see Fig. 1).

Due to the observed non-uniform distribution in proper inclination, in our generation of Quasi Random populations in this region we use only three bins in  $\sin(I_p)$  (see Table 1). These bins have been chosen in such a way that each of them covers one of the three different dominions in inclination.

The stalactite diagram is shown in Fig. 7. The overall structure seems to consist of three major branches merging together at high distance levels. This might reflect the particular distribution of the objects in proper inclination. At least one additional, well separated and very compact group, however, is also clearly visible as a very deep stalactite branch. The lowest-numbered member of this family is (780) Armenia. In total, we identified 17 asteroid families, 21 clumps and 4 clusters in this region.

The region is dominated by two large families, associated to asteroids (31) Euphrosyne, and (181) Eucharis.

Among our nominal families, those of Euphrosyne, Alauda and Moravia had been already identified by other authors (Foglia and Masi, 2004; Gil-Hutton, 2006). It is interesting to note that asteroid (702) Alauda is known to be a binary system (Margot and Rojo, 2007). The companion is much smaller than the primary (Rojo and Margot, 2011), suggesting (but this is only a conjecture) that it might represent captured ejecta from a collision.

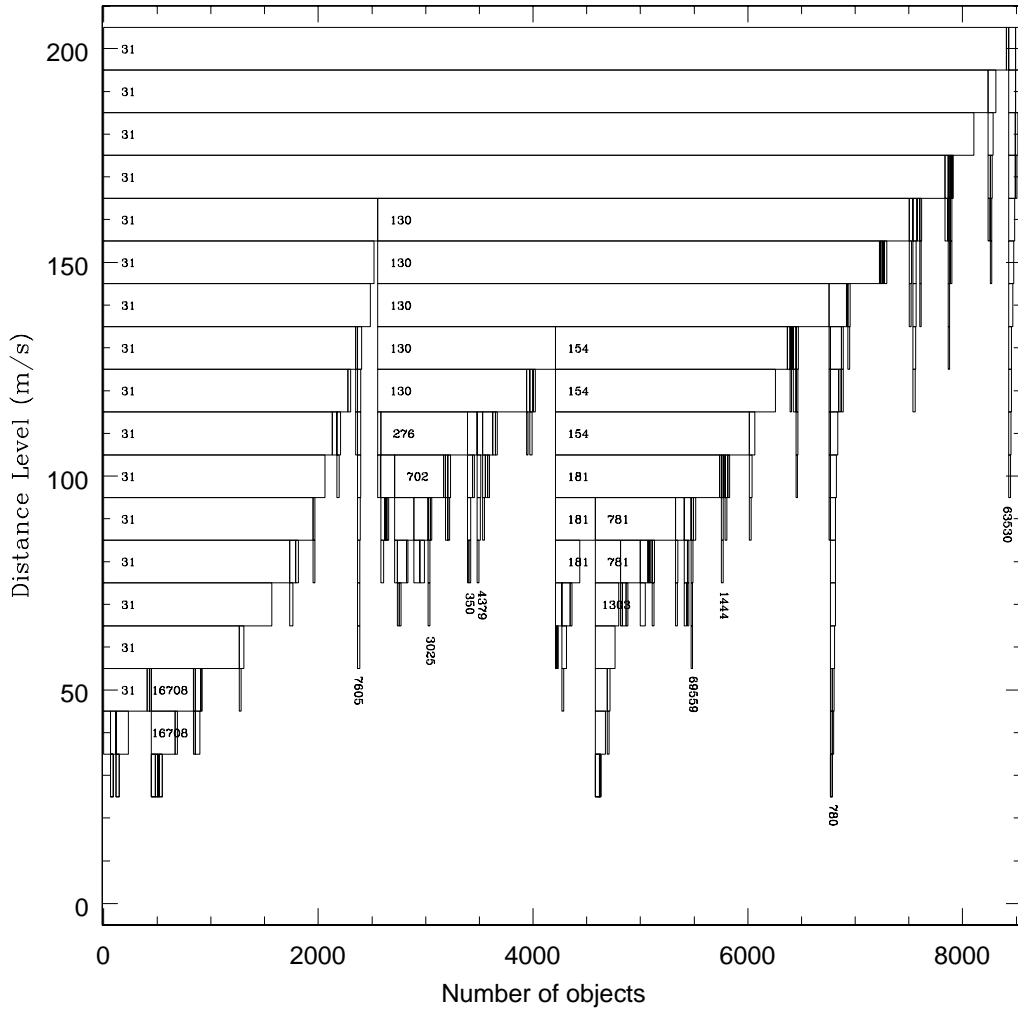


Fig. 7. The stalactite diagram for the outer zone ( $N_{crit}=14$ ,  $QRL = 90$  m/s). As in Figs. 5 and 6, at each distance level only groupings having at least  $N_{crit}$  members are shown.

Our families of Eucharis, Pannonia, Filipenko and Snelling had been classified as clumps in previous investigations (Gil-Hutton, 2006).

The third largest family in the outer zone is associated to asteroid (781) Kartvelia. This group is a newly discovered family, but we note that it merges with Eucharis just 10 m/s above the critical QRL. In addition, nine new families have been identified in this region. These are Ornamenta, Armenia, Vassar, Higson, Zhvanetskij, 1995SR<sub>1</sub>, 1994VD<sub>7</sub>, 1998HV<sub>32</sub> and 1997UG<sub>5</sub>.

Former asteroid families Weber and (16708) 1995SP<sub>1</sub> (Gil-Hutton, 2006), are now parts of the Hirose clump and the Euphrosyne family, respectively. We did not find any significant groupings associated with asteroids (1303) Luthera and (6051) Anaximenes, which were proposed to be families by Gil-Hutton (2006).

## 4 SDSS colors and Spectroscopic data

According to previous analyzes available in the literature, it turns out that, as a general rule, the members of each family tend to share similar spectral characteristics (e.g. Bus, 1999; Florczak et al., 1999; Lazzaro et al., 1999; Ivezić et al., 2002; Cellino et al., 2002). Spectral properties of families can thus be used to complement the results of our HCM analysis of proper elements. In particular, spectral information may be used both to identify possible family interlopers as well as to identify objects that might be candidate family members, although they are not included in nominal member lists derived by proper element information only (Migliorini et al., 1995; Milani et al., 2010). Therefore, we have carried out an analysis of available SDSS colors for the asteroids of our sample.

For the purpose of deriving reliable inferences about asteroid surface compositions, multi-band photometry is not as precise as spectroscopy. However, SDSS data are very important, because this survey includes about two orders of magnitude more objects than available spectroscopic catalogs. Recently, SDSS data were used by Roig and Gil-Hutton (2006) to identify possible basaltic (*V*-type) asteroids, and by Parker et al. (2008) to analyze characteristics of the classical main belt families. Here, we use the fourth release of the SDSS Moving Object Catalog (MOC 4) to analyze the color distribution properties among different asteroid groups identified in this work. In cases when spectroscopic data are also available, these are exploited to reach more reliable conclusions. In particular, we used taxonomic/spectral classifications based on SMASS I (Xu et al., 1995), SMASS II (Bus and Binzel, 2002) and S3OS2 (Lazzaro et al., 2004) surveys. In addition, the much older ECAS survey (Zellner et al., 1985; Tholen, 1989) was also used whenever possible.

Nesvorný et al. (2005) showed that the SDSS MOC is a useful, self-consistent dataset to study general statistical variations of colors of asteroids in the main belt, but caution is required to interpret colors in individual cases. The above authors used an automatic algorithm of Principal Component Analysis (PCA) to analyze SDSS photometric data and to sort the objects into different taxonomic classes.

In particular, PCA can be used to derive linear combinations of the five SDSS colors, in order to maximize the separation between a number of different taxonomic classes in SDSS data.<sup>7</sup>

According to Nesvorný et al. (2005) (see also Ivezić et al., 2001) the first two principal components can be used to distinguish among big taxonomic complexes such

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<sup>7</sup> In principle, this kind of analysis can also be made using method adopted by Ivezić et al. (2002) and Parker et al. (2008). They used  $(a^*, i - z)$  instead of  $(PC_1, PC_2)$  plane, where  $a^*$  is calculated according to the following relation:

$$a^* = 0.89(g - r) + 0.45(r - i) + 0.09(g - i) - 0.57 .$$

as  $S$ ,  $C$ , or  $X$  (see Bus and Binzel, 2002, for definitions of different taxonomic complexes/classes). These complexes are found to occupy different locations in the  $(PC_1, PC_2)$  plane.

Following the same procedure, we obtained the relations which define the two principal components for our sample of asteroids, which includes 3689 high-inclination objects that are present in the SDSS MOC 4. The resulting relations are:

$$PC_1 = -0.337(u - g) + 0.470(g - r) + 0.618(g - i) + 0.533(g - z) , \quad (2)$$

$$PC_2 = -0.654(u - g) + 0.489(g - r) - 0.305(g - i) - 0.491(g - z) . \quad (3)$$

where  $u, g, r, i, z$  are the measured fluxes in five SDSS bands after correction for solar colors; for the values of solar colors see Ivezić et al. (2001).

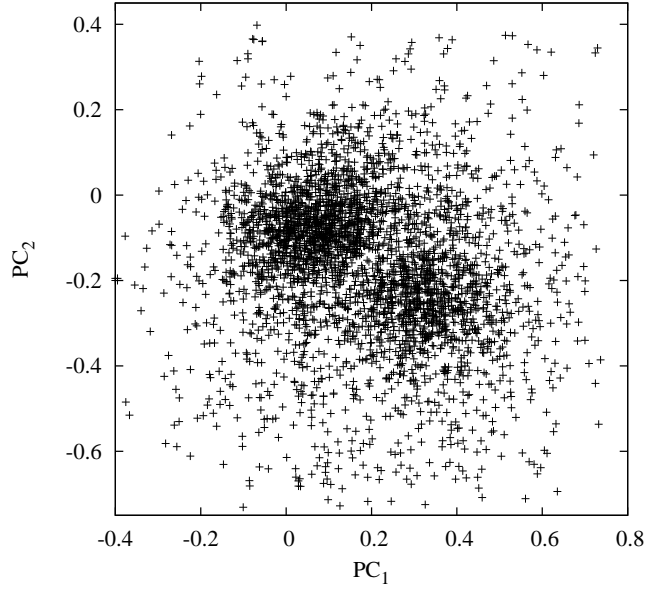


Fig. 8. The distribution of high-inclination asteroids included in both our sample of proper elements and in the SDSS MOC 4, plotted in the  $(PC_1, PC_2)$  plane.

In Fig. 8 we plot our sample in the  $(PC_1, PC_2)$  plane. Two slightly separated, very dense regions, immersed in a more sparse background, can be easily recognized. This general behavior was already found by Nesvorný et al. (2005) and Parker et al. (2008), who found an association of the two major groups with different spectral complexes. Following their example, we plot in Fig. 9 the positions of asteroids with known spectral types in the plane of our principal components. From the figure we see that two groups visible in Fig. 8 correspond to  $S$  (bottom right) and  $C/X$  (top left) complexes.

The  $S$  complex appears to be fairly well separated from the  $C/X$  complex. The situation is worse for the  $X$  and  $C$  types, which tend to overlap each other in our PC plane. An  $X$  and  $C$  overlapping, although slightly less evident, was also found by Nesvorný et al. (2005).

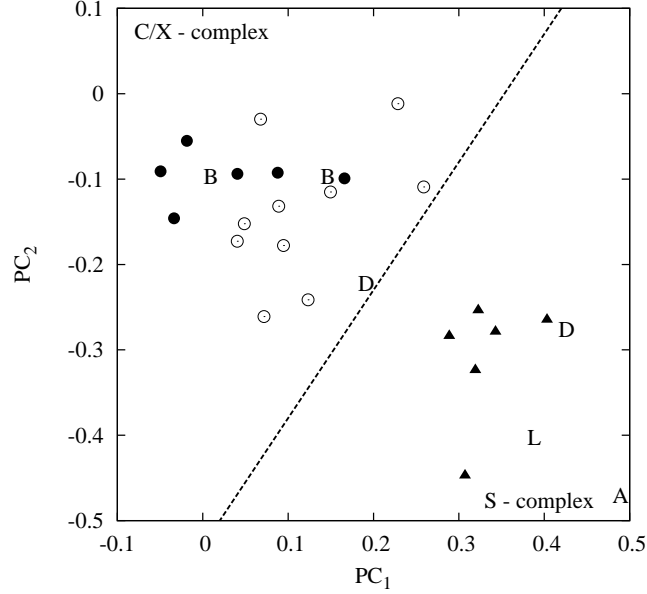


Fig. 9. Locations of the twenty eight high-inclination asteroids included in the SDSS MOC 4 having known spectral types in the plane of the principal components obtained according to the Eqs. (2) and (3). Filled circles represent *C*-type, open circles *X*-type and triangles are *S*-type asteroids. The locations of two *B*, two *D*, one *A*, and one *L*-type asteroids are also shown. The separation between *S* and *C/X* spectral complexes is quite clear. The approximate border is shown by the dashed line. Unfortunately, it is not possible to separate reliably the *C* and *X* complexes, because they are mixed together making it very difficult to find a clear distinction between them.

In Fig. 9 the *C* complex is typically located at somewhat smaller values of  $PC_1$  and higher values of  $PC_2$  with respect to *X* complex objects, but a considerable mixing of the two complexes is present. As a consequence, we are generally able to distinguish only among *S* and *C/X* taxonomic complexes, although some comments on possible distinctions between *C* and *X* are given in some cases discussed below.

As a first step, we examined the resulting abundance of *S* and *C/X* asteroids in the three zones defined in our HCM analysis, in order to analyze the variation in relative abundance of the main taxonomic complexes as a function of heliocentric distance. The results are given in Table 5.

Table 5

The fraction of objects belonging to the *S* and *C/X* taxonomic complexes according to their SDSS MOC 4 colors, as found in each of three zones, as well as in the total sample of high-inclination asteroids.

Complex	Inner zone	Interm. zone	Outer zone	Total
<i>S</i>	464 (72%)	537 (54%)	368 (21%)	1369 (40%)
<i>C/X</i>	177 (28%)	460 (46%)	1408 (79%)	2045 (60%)
Total	641	997	1776	3414

As can be seen, and not unexpectedly, the *S* complex dominates in the inner zone,

whereas the  $C/X$  complex is dominant in the outer zone. In the middle region the two complexes have similar abundances. Although a somewhat larger fraction of  $S$  asteroids could be expected in the intermediate zone, our results are in general agreement with current knowledge about the abundance of different taxonomic classes as a function of heliocentric distance (e.g. Bus and Binzel, 2002; Mothé-Diniz et al., 2003). However, we found a significantly larger abundance of  $S$  asteroids in the outer zone with respect to recent results by Carvano et al. (2010), who performed a similar study for low-inclination asteroids in the main belt. According to Mothé-Diniz et al. (2003), differences in the abundance of different taxonomic complexes across the main belt exist between low- and high-inclination asteroids. These authors also found that the abundance of  $S$ -class asteroids is significantly affected by the presence of asteroid families. Our results confirm these findings, although it is not clear to us whether observational biases acting against the discovery of high-inclination, low-albedo asteroids in the outer belt could also play an important role. In any case, we find that a major contribution to the relative abundance of  $S$  class asteroids in the outer belt at high orbital inclination, is due to the presence of one single, large family, having as its lowest-numbered member the asteroid (181) Eucharis (see also the discussion below).

Having the values of principal components, calculated using Eqs. (2) and (3), we could compute the average values of  $PC_1$  and  $PC_2$  for all families identified by HCM for which at least 5 members are included in the SDSS MOC 4. The corresponding values are listed in Table 6.

Table 6: The list of the asteroid families, clumps and clusters, identified in this work, with available color data in the SDSS MOC 4. Only groups with at least five members included in the color survey are shown. For each family, the Table gives: family name; number  $N$  of members; number  $N_{SDSS}$  of members observed by SDSS; the values of the principal components along with their standard deviations; taxonomic complex according to the values of the SDSS principal components.

Name	$N$	$N_{SDSS}$	$PC_1$	$\sigma_{PC_1}$	$PC_2$	$\sigma_{PC_2}$	Taxonomy
Inner belt							
(25) Phocaea	1694	288	0.265	0.171	-0.251	0.194	$S$
Intermediate belt							
(2) Pallas	57	9	-0.038	0.133	0.017	0.097	$C/X$
(148) Gallia	113	22	0.290	0.165	-0.330	0.175	$S$
(480) Hansa	839	162	0.291	0.141	-0.230	0.177	$S$
(686) Gersuind	207	40	0.390	0.118	-0.279	0.145	$S$
(729) Watsonia	139	31	0.319	0.154	-0.340	0.188	$S$
(945) Barcelona	600	91	0.227	0.152	-0.182	0.194	$C/X$
(1222) Tina	89	17	0.120	0.217	-0.130	0.171	$C/X$
(4203) Brucato	46	11	0.056	0.104	-0.155	0.121	$C/X$
(4404) Enirac	52	6	0.157	0.118	-0.217	0.132	$C/X$
(10000) Myriostos	73	14	0.183	0.190	-0.117	0.280	$C/X$
(29905) 1999 $HQ_{11}$	28	9	0.255	0.201	-0.249	0.161	$S$
(40134) 1998 $QO_{53}$	24	6	0.200	0.255	-0.293	0.149	$S$
(62074) 2000 $RL_{79}$	33	9	0.306	0.080	-0.183	0.150	$S$



(108696) 2001 $OF_{13}$	36	5	0.167	0.087	-0.273	0.147	$C/X$
Outer belt							
(31) Euphrosyne	2066	323	0.087	0.162	-0.045	0.174	$C/X$
(181) Eucharis	778	149	0.390	0.229	-0.344	0.296	$S$
(350) Ornamenta	93	14	0.025	0.179	-0.056	0.122	$C/X$
(702) Alauda	179	46	0.026	0.138	-0.096	0.116	$C/X$
(780) Armenia	76	13	0.103	0.137	-0.063	0.170	$C/X$
(781) Kartvelia	232	49	0.293	0.109	-0.150	0.150	$S$
(1101) Clematis	16	5	-0.025	0.056	-0.174	0.179	$C/X$
(2967) Vladisvyat	74	11	0.036	0.174	-0.120	0.230	$C/X$
(3025) Higson	17	5	0.062	0.106	-0.065	0.110	$C/X$
(5931) Zhvanetskij	64	20	0.058	0.166	-0.151	0.165	$C/X$
(19254) 1994 $VD_7$	26	6	0.042	0.055	-0.037	0.118	$C/X$
(24440) 2000 $FB_1$	16	7	0.037	0.142	-0.079	0.082	$C/X$
(25295) 1998 $WK_{17}$	19	6	0.145	0.234	-0.104	0.130	$C/X$
(28884) 2000 $KA_{54}$	18	7	0.069	0.102	-0.069	0.150	$C/X$
(58892) 1998 $HP_{148}$	18	7	0.060	0.180	-0.214	0.220	$C/X$

We found that most families in each zone belong to the dominant spectral type/complex. This can be better appreciated in Fig. 10, in which we show the locations of families, clumps and clusters in the  $(PC_1, PC_2)$  plane.

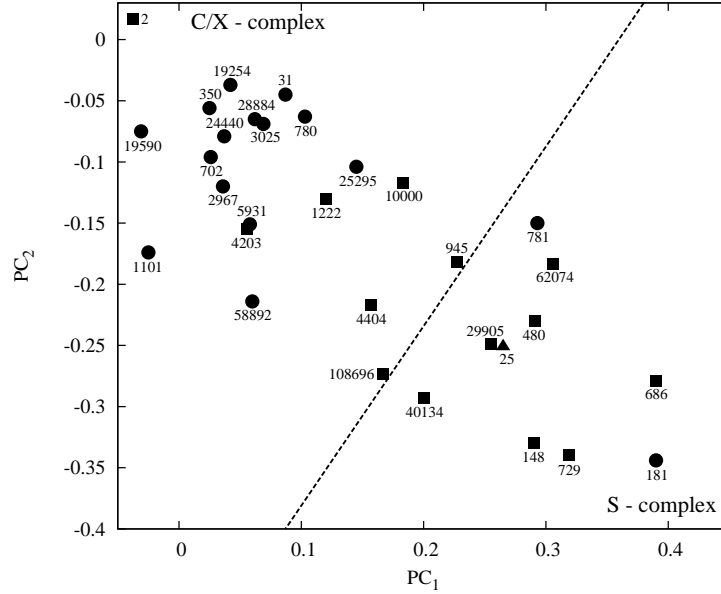


Fig. 10. Locations of the asteroid families, clumps and clusters identified in this work, in the  $(PC_1, PC_2)$  plane. The groups located in the inner, intermediate and outer zone are shown as triangles, squares and circles, respectively. The dashed inclined line represents approximately the border between the  $S$  and  $C/X$  complexes.

In the inner zone, as expected, all groups belong to the *S* complex. However, only the Phocaea family has at least five members included in the SDSS data-set.

More in particular, color data are available for 288 members of the Phocaea family. About 80% of these belong to the *S*-complex. Spectral types (derived from spectroscopy) are available for 35 asteroids that belong to the Phocaea family. Most of them are the *S*-class with only three exceptions, which correspond to likely interlopers. The asteroids (326) Tamara and (1963) Bezovec are *C*-class, while asteroid (1318) Nerina is an *X*-class.

In the intermediate zone, the numbers of families belonging to the *S* and *C/X* complexes are the same. Seven families include *S* members, and seven are *C/X*. This is in a good agreement with the relative abundance of these two complexes in this region of the belt (see Table 5). The number of family members with known spectral type is very limited. However, in most cases these data are consistent with derived SDSS colors. Let us discuss here only a few exceptions.

The Gersuind family includes two members with known spectral type, namely asteroid (686) Gersuind itself, which is *S* class, and asteroid (1609) Brenda which is classified as *D* type. The obtained values of  $PC_1$  suggest that members of this family should be *S*-type asteroids. This is in agreement with the spectral classification of (686) Gersuind. As for the *D* classification of (1609) Brenda, the *D* class turns out to be a subgroup of the *S* complex in our Principal Components analysis. Of course, we are aware that *S* class asteroids are expected to be quite distinct from *D* class objects in terms of thermal history and composition. *D* class asteroids have featureless and very reddish spectra, and are most common among Jupiter Trojans, whereas they are relatively less common in the main belt. A numerical simulation performed by Levison et al. (2009) showed that these bodies may have originated from trans-Neptunian region as a result of the violent dynamical evolution of the giant-planet orbits as suggested by the so-called Nice model (Tsiganis et al., 2005; Morbidelli et al., 2005; Gomes et al., 2005). Interestingly, results of Levison et al. (2009) suggest an inner boundary for this type of objects around 2.6 AU, while asteroid (1609) Brenda has a semi-major axis of about 2.58 AU. In any case, we do not rule out the possibility that either the taxonomic classification of (1609) Brenda could be wrong, or it may be an interloper in the Gersuind family.

Reflectance spectra are available also for two members of the Myriostos family. The asteroid (344) Desiderata turns out to be a *C*, while (1246) Chaka belongs to the *S*-class. According to SDSS colors, the members of this family belong to the *C/X* complex, in agreement with spectral evidence for (344) Desiderata. Therefore, it is likely that (1246) Chaka is an interloper. Finally, the *C*-class asteroid (3037) Alku is probably an interloper within the *S*-type (29905) 1999*HQ*<sub>11</sub> family.

At least in some cases, very similar values of the principal components among two groups might suggest a common origin, like, for example, in the case of Higson family and the (28884) 2000*KA*<sub>54</sub> clump. They are very close in terms of principal components, and merge at a distance cut-off of 110 m/s in the space of proper elements. This might well be a first example of spectroscopic confirmation of a

genetic relation between a family and an associated clump. However, available data are not sufficient to draw definite conclusion in this respect, but we think that new observations can provide interesting results in the future.

In the outer region almost all identified groups belong to the  $C/X$  complex. There are only two exceptions. The Eucharis and Kartvelia families belong to the  $S$  complex. Interestingly, the Eucharis family is located in a quite peculiar location at the far edge of the  $S$  dominion in the  $(PC_1, PC_2)$  plane (see Fig. 10), ruling out any possibility that it might have anything to do with the  $C/X$  complex. Although we cannot distinguish clearly between the  $C$  and  $X$  complex in our SDSS analysis, some indication about a preferred location for the  $C$  complex can be drawn. In particular, most families in the outer belt seem to cluster around a single sub-dominion of the  $(PC_1, PC_2)$  portion of plane occupied by the  $C/X$  complexes. Available spectral types are consistent with this conclusion. The asteroids (350) Ornamenta and (780) Armenia belong to the  $C$  class. Moreover, two members of Euphrosyne and four members of Alauda that have available spectral types, are all consistent with the  $C$  complex.

The situation of the Eucharis family is in some way unusual. Four members of this family have known spectral types. Two of them, including asteroid (181) Eucharis itself, are  $X$ -class, whereas two are  $C$ -class. None of these objects is consistent with an  $S$ -class classification inferred for this family from our analysis of the SDSS colors for the members of this family.

Since it belongs to the  $S$  spectral class, which is relatively rare in the outer belt, the Eucharis family can be clearly distinguished from nearby background asteroids. According to available SDSS data, about 25% of the Eucharis family members identified in our analysis would be interlopers. This is an exceedingly large fraction with respect to usual situations (Miglierini et al., 1995; Parker et al., 2008) and might be an indication of the presence of another separate family overlapping with Eucharis. Among the suspected interlopers there are several large asteroids such as (285) Regina, (746) Marlu, (1035) Amata, and (29943) 1999JZ<sub>78</sub>. The situation is made even more complicated by the fact that the Eucharis family is located not far from some well known low-inclination families including (137) Meliboea and (1400) Tirela. Further investigations, that we postpone for a future paper, are necessary to address these questions.

## 5 Summary and Conclusions

In this paper a comprehensive search for asteroid families among the population of high-inclination asteroids has been presented. The search has been performed by applying the standard Hierarchical Clustering Method to a sample of 10,265 numbered objects for which synthetic proper elements were taken from the *AstDys* web site. To these, we added other 8,295 multi-opposition objects for which we computed synthetic proper elements. We included in our sample only asteroids

having sine of proper inclination greater than 0.295.

We considered three zones corresponding to three different intervals of proper semi-major axis (inner, intermediate and outer region). We used the HCM to identify families in each zone. In doing so, we applied HCM in generally the same way as it was applied in the past for family searches among the low-inclination population. However, we also introduced some improvements in the procedure, to achieve a more reliable and robust classification, mainly for what concerns family membership. We also make a clear distinction between highly reliable groupings, that we call families, and more uncertain ones, that we call clumps. In addition, we call clusters some very compact groupings for which the number of objects is still low, but could increase in the future, as more and more objects will be discovered by observational surveys. The best example of cluster we found is a very compact eight-members grouping including (5438) Lorré.

We took advantage of available SDSS MOC 4 color data to improve family membership reliability and identify likely family interlopers. Using Principal Component Analysis, we classified all families into *S* or *C/X* taxonomic complexes. We found that taxonomical distribution of families matches very well a systematic variation of asteroid spectral type with respect to heliocentric distance. Some exceptions exist, however, a very interesting case being that of the Eucharis family.

Asteroid families identified here provide a wide range of opportunities for possible future studies related to high-inclination asteroids. Our results are only the first step to fully understanding collisional evolution of this part of asteroid belt. There is a lot of work that should be done. For example, to study dynamical characteristics of proposed families, to estimate their ages, to find size-frequency distributions (SFDs) of family members, to estimate the size of parent bodies, etc. These results should be then compared to those obtained for the classical main belt, what would allow us to understand how differently these two populations evolved. Also, typical relative velocity among the high-inclination asteroids is about 11 km/s (Gil-Hutton, 2006), while in the classical belt it is only about 5 km/s (Bottke et al., 1994). Thus, it is interesting to see how SFDs of high-inclination families fit in numerical experiments (Michel et al., 2003; Durda et al., 2007).

Among the individual cases as a particularly interesting to study we highlight the possibly interplay among Eucharis, Meliboea and Tirela families. Different chronology methods (Vokrouhlický et al., 2006; Novaković et al., 2010; Cachucho et al., 2010) could be successfully applied to these groupings.

Some of the studies mentioned above are already possible with existing data, while some others will be possible in the near future. Different observational surveys will provide physical characteristics (e.g. albedos, rotational periods, diameters, spectral types) for many asteroids, including these on highly inclined orbits. Among these surveys let us mention here one just finished, Wide-Field Infrared Survey Explorer (WISE), and one planned to be launched in 2013, Global Astrometric Interferometer for Astrophysics (GAIA). Other surveys, like this presented recently by Terai and Itoh (2011), could provide valuable data as well.

The results of this investigation open also perspectives for new, dedicated observing campaigns. In particular, we mention the interesting case of the Pallas family, which certainly deserves some further spectroscopic investigations in the visible and near-IR, as already suggested by Clark et al (2010). Moreover, polarimetric observations of the Anacostia and Watsonia families in the middle region, might likely lead to discover new examples of “Barbarians”. Observations of small compact clusters like Lorre might be highly welcome as well.

## Acknowledgments

We would like to thank David Nesvorný and Valerio Carruba, the referees, for their useful comments and suggestions which helped to improve this article. The work of B.N. and Z.K. has been supported by the Ministry of Education and Science of Serbia, under the Project 176011.

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